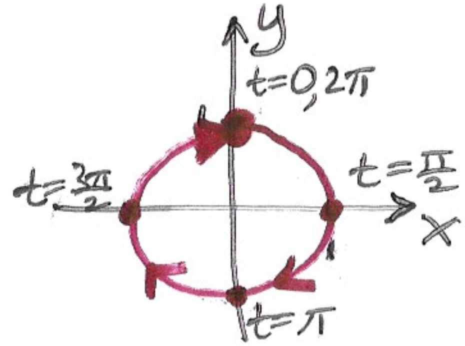
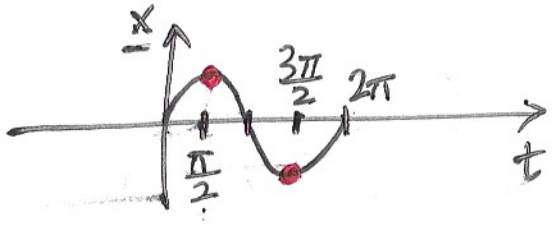
11.6

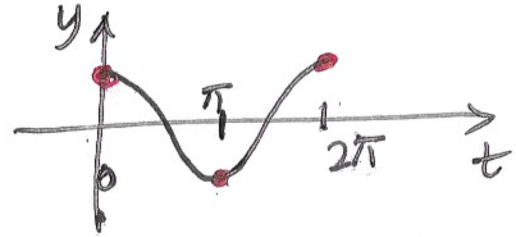
SKETCH $x = \sin t$, $y = \cos t$, $t \in [0, 2\pi]$



① SKETCH BOTH ~~CURVES~~ EQUATIONS SEPARATELY

(t ON HORIZONTAL AXIS





② LOCATE MAX/MIN OF x, y + RECORD t-VALUES

③ DRAW NUMBER LINE FOR t + MARK OFF

x		INCR →		DECR ←		DECR ←		INCR →
y		DECR ↓		DECR ↓		INCR ↑		INCR ↑
POINTS	(0,1)		(1,0)		(0,−1)		(−1,0)		(0,1)
t	0	↘	$\frac{\pi}{2}$	↙	π	↖	$\frac{3\pi}{2}$	↗	2π

← POINTS

← DIRECTIONS BETWEEN POINTS

$x = \sin t$	$x = \sin 0 = 0$	$x = \sin\frac{\pi}{2} = 1$	$x = 0$	$x = -1$	$x = 0$
$y = \cos t$	$y = \cos 0 = 1$	$y = \cos\frac{\pi}{2} = 0$	$y = -1$	$y = 0$	$y = 1$

ELIMINATE THE PARAMETER FOR $x = \sin t$, $y = \cos t$, $t \in [0, 2\pi]$

$t = \sin^{-1} x \neq \frac{1}{\sin x}$

or $t = \arcsin x$

$y = \cos(\underbrace{\arcsin x})$ GRAPH ?

RANGE OF $\arcsin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

IE. Q_1, Q_4

RANGE OF $\cos(\arcsin x) > 0$

$y \in [0, 1]$

DON'T USE INVERSE TRIG TO ELIMINATE PARAMETER

INSTEAD USE IDENTITIES

$\sin^2 t + \cos^2 t = 1$

$x^2 + y^2 = 1$ ← CIRCLE CENTER $(0,0)$ RADIUS 1

$x = \sin t$
$y = \cos t$
$\downarrow$
$x^2 + y^2 = 1$

CIRCLE
CENTER (0,0)
RADIUS 1

STARTING AT TOP
CLOCKWISE

$x = \cos t$
$y = \sin t$
$\downarrow$
$x^2 + y^2 = 1$

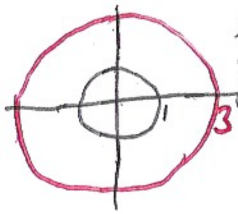
CIRCLE
CENTER (0,0)
RADIUS 1

START ON RIGHT SIDE
COUNTERCLOCKWISE

$x = -\sin t$, $y = \cos t$ $\quad$ $x = -\sin t$, $y = -\cos t$ $\quad$ $x = \sin t$, $y = -\cos t$ $\quad$ . . .

SAME CIRCLE
DIFFERENT STARTING POINTS & DIRECTIONS
$\downarrow$
ORIENTATION

CIRCLE CENTER (0,0)
RADIUS $= \mathbf{3} = R$
$\downarrow$
$x = R\sin t$
$y = R\cos t$
OR
$x = R\sin\theta$
$y = R\cos\theta$
OR
$x = R\sin s$
$y = R\cos s$



$x = \sin t$, $y = \cos t$ $\longrightarrow$ $x = 3\sin t$, $y = 3\cos t$

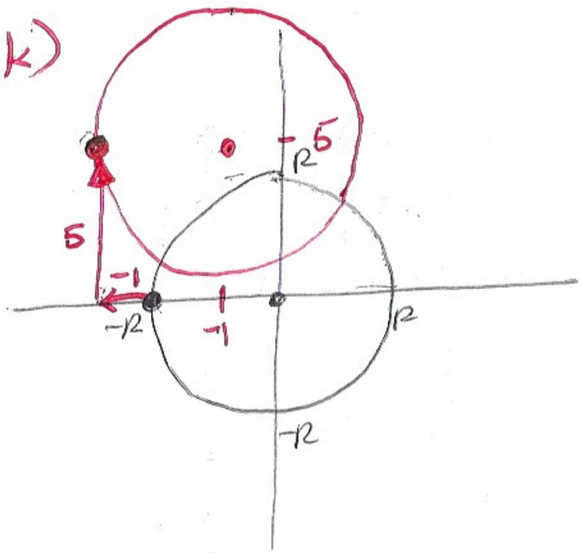
(x,y) ON BIG CIRCLE ARE 3 *
(x,y) ON SMALL CIRCLE

t, θ, s ARE NAMES OF PARAMETERS

CIRCLE CENTER $(-1,5)=(h,k)$

RADIUS $=R$

$$\boxed{\begin{aligned} x &= h+R\sin t \\ y &= k+R\cos t \end{aligned}}$$



$x=R\sin t$
$y=R\cos t$

CENTER $(0,0)$

$\downarrow$

$x=R\sin t-1$
$y=R\cos t+5$

OR

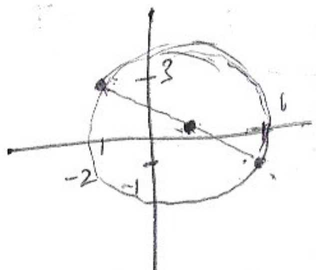
$x=-1+R\sin t$
$y=5+R\cos t$

FIND PARAMETRIC EQUATIONS FOR CIRCLE WITH A DIAMETER WITH END POINTS $(-2,3)$ AND $(6,-1)$

$x=h+R\sin t$
$y=k+R\cos t$

$\downarrow$

$x=2+2\sqrt{5}\sin t$
$y=1+2\sqrt{5}\cos t$



CENTER = MIDPOINT OF ENDPOINTS OF ANY DIAMETER

$=\left(\frac{1}{2}(-2+6),\frac{1}{2}(3+-1)\right)$

$=\boxed{(2,1)}$

RADIUS $=\frac{1}{2}$ DIAMETER

$=\frac{1}{2}\sqrt{(6--2)^2+(-1-3)^2}$

$=\frac{1}{2}\sqrt{8^2+(-4)^2}$

$=\frac{1}{2}\sqrt{4^2(2^2+1^2)}$

$=\frac{1}{2}\cdot 4\sqrt{5}$

$=\boxed{2\sqrt{5}}$

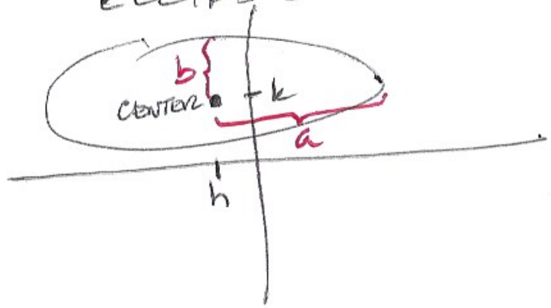
$$x = h + R\sin t$$
$$y = k + R\cos t$$
$$\downarrow$$
$$\sin t = \frac{x-h}{R}$$
$$\cos t = \frac{y-k}{R}$$

$$\sin^2 t + \cos^2 t = 1$$
$$\left(\frac{x-h}{R}\right)^2 + \left(\frac{y-k}{R}\right)^2 = 1$$
$$\frac{(x-h)^2}{R^2} + \frac{(y-k)^2}{R^2} = 1$$
$$(x-h)^2 + (y-k)^2 = R^2$$

CIRCLE CENTER (h,k)

RADIUS $\sqrt{R^2} = R$ ✓

ELLIPSES



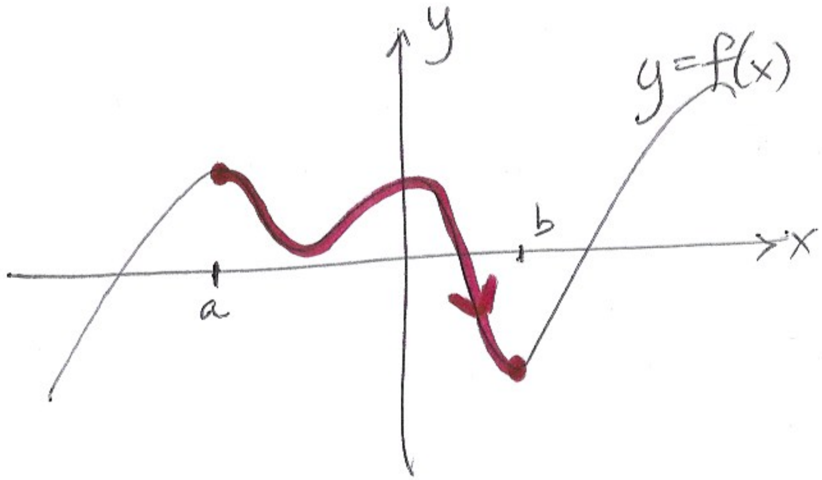
$$x = h + a\sin t$$
$$y = k + b\cos t$$
$$\longrightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

CONSIDER $x = \sinh t$, $y = \cosh t$ $\longrightarrow$ $\cosh^2 t - \sinh^2 t = 1$

$$y^2 - x^2 = 1 \longrightarrow \text{HYPERBOLA}$$

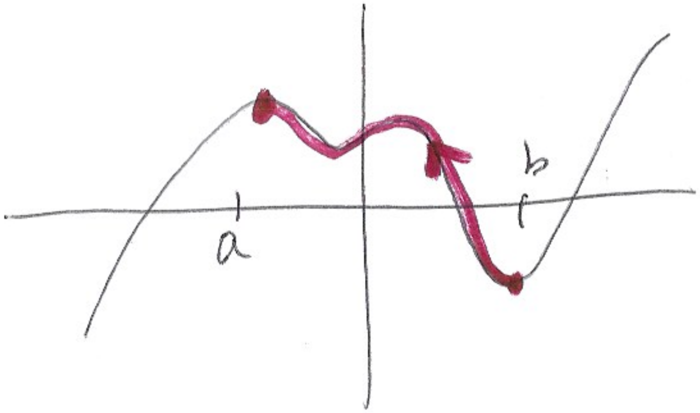
CONSIDER PART OF GRAPH OF $y=f(x)$ FROM $x=a$ TO $x=b$

$(a<b)$



$x=t$

$y=f(t)$ $\quad t\in[a,b]$



$x=t$

$y=f(t)$ $\quad t\in[b,a]$

$b \nless a$